Block Diagram Manipulation
MECH 412 - *System Dynamics and Control*

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- The Laplace transform enables

$$\dot{q}(t) + aq(t) = bu(t), \quad \forall t \geq 0, \tag{1}$$
$$y(t) = q(t), \tag{2}$$

to be written as, or abstracted to

$$y(s) = \underbrace{\left(\frac{b}{s+a}\right)}_{G(s)} u(s). \tag{3}$$

- As shown in Figure 1, (3) can be written in *block diagram form*.

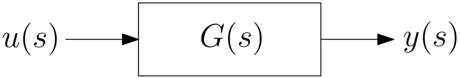

Figure 1: Block diagram representation of $y(s) = G(s)u(s)$.

- In general, $G(s)$ could be any transfer function, not just a first-order transfer function, as in (3).
- When many systems interact, it's often convenient to manipulate the *block diagram* describing their interconnection.
- The three most common block diagram interconnections are presented next: *series*, *parallel*, and *negative feedback* interconnections.

Series Interconnection

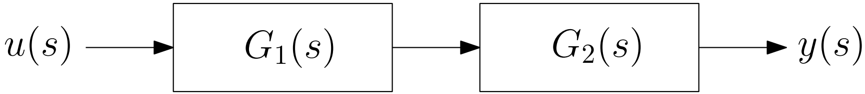

Figure 2: Series interconnection of $G_1(s)$ and $G_2(s)$.

- Consider the series interconnection in Figure 2.
- Mathematically,

$$
\begin{aligned}
y(s) &= G_2(s)u_2(s), \quad u_2(s) = y_1(s) = G_1(s)u(s),\\
y(s) &= G_2(s)\left(G_1(s)u(s)\right)\\
y(s) &= \left(G_2(s)G_1(s)\right)u(s).
\end{aligned}
$$

- Thus, the block diagram of Figure 2 is equivalent to the block diagram of Figure 3.

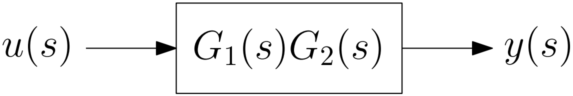

Figure 3: Series interconnection of $G_1(s)$ and $G_2(s)$ manipulated to one block.

Parallel Interconnection

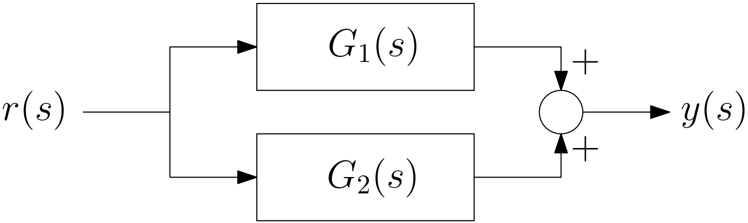

Figure 4: Parallel interconnection of $G_1(s)$ and $G_2(s)$.

- Consider the parallel interconnection in Figure 4.
- Mathematically,

$$
\begin{aligned}
y_1(s) &= G_1(s)u_1(s) = G_1(s)r(s), \\
y_2(s) &= G_2(s)u_2(s) = G_2(s)r(s), \\
y(s) &= y_1(s) + y_2(s).
\end{aligned}
$$

- It follows that

$$
\begin{aligned}
y(s) &= y_1(s) + y_2(s) \\
&= G_1(s)r(s) + G_2(s)r(s) \\
&= (G_1(s) + G_2(s))\, r(s).
\end{aligned}
$$

- Thus, the block diagram of Figure 4 is equivalent to the block diagram of Figure 5.

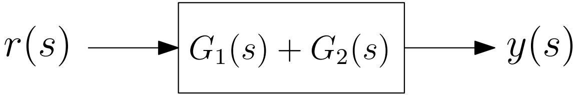

Figure 5: Parallel interconnection of $G_1(s)$ and $G_2(s)$ manipulated to one block.

Negative Feedback Interconnection

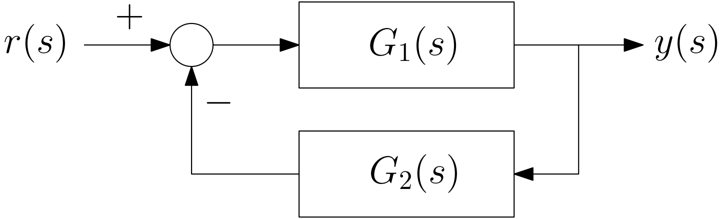

Figure 6: Negative feedback interconnection of $G_1(s)$ and $G_2(s)$.

- Consider the negative feedback interconnection in Figure 6.
    - Assume the negative feedback interconnection is well posed. Well posedness will be discussed in later.
- Mathematically,

$$\begin{aligned} y_1(s) &= G_1(s)u_1(s), \\ y_2(s) &= G_2(s)u_2(s), \\ y_1(s) &= y(s) = u_2(s), \\ u_1(s) &= e(s) = r(s) - y_2(s). \end{aligned}$$

- It follows that

$$
\begin{aligned}
y(s) &= G_1(s)e(s) \\
&= G_1(s)(r(s) - y_2(s)) \\
&= G_1(s)\left(r(s) - G_2(s)u_2(s)\right) \\
&= G_1(s)\left(r(s) - G_2(s)y(s)\right) \\
&= G_1(s)r(s) - G_1(s)G_2(s)y(s).
\end{aligned}
$$

- Collecting like-terms in $y(s)$,

$$
\begin{aligned}
y(s) + G_1(s)G_2(s)y(s) &= G_1(s)r(s), \\
\left(1 + G_1(s)G_2(s)\right) y(s) &= G_1(s)r(s), \\
y(s) &= \left(\frac{G_1(s)}{1 + G_1(s)G_2(s)}\right) r(s).
\end{aligned}
$$

- Thus, the block diagram of Figure 6 is equivalent to the block diagram of Figure 7.

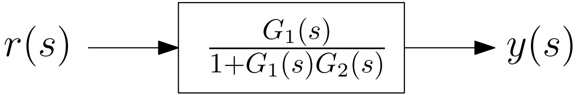

Figure 7: Feedback interconnection of $G_1(s)$ and $G_2(s)$ manipulated to one block.

References

These slides are based on notes from Prof. Chris Damaren (UTIAS) and [1].

[1] L. Qiu and K. Zhou, *Introduction to Feedback Control*. Upper Saddle River, NJ: Prentice-Hall, Inc., 2010.